# Maximizing the Capacity of Ultra-Long Haul Submarine Systems

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Abstract—The capacity of ultra-long haul (ULH) submarine systems is ultimately limited by the electrical power that can be delivered to the submerged amplifiers within the repeaters. An excessive power feed voltage will cause damage to the cable and joints and so the system should be designed to maximize the power available per repeater using the available voltage. Furthermore, the optical system design must use the available power efficiently in order to maximise the system capacity. Here, both electrical and optical design methodologies are considered which achieve this aim. It is found that the ideal span loss is in the range 8 - 10 dB and that the system should be operated in the linear regime of the fibre, obviating the need for high effective area fibres. Furthermore it is shown that higher order modulation formats are inefficient in ULH systems as sufficient bandwidth is available and standard PM-QPSK allows for greater capacities. In this respect, spectral efficiency is a poor metric to assess such systems. Through optimization, it is shown that transpacific systems of up to 250 Tb/s over 10 SDM channels are feasible using amplifier bandwidths of 70 nm.

#### I. INTRODUCTION

One of the fundamental constraints in the design of ULH undersea fibre-optic systems is the electrical powering of the submarine plant. The repeaters are electrically connected in series and so for a long system, we have to ensure that the potential difference across the cable and repeaters does not exceed the maximum rating of the cable and joints, which is typically 12 kV. We also need to keep the current in check in order to minimise ohmic losses in the cable and this in turn limits the power that is deliverable to the repeaters. Until relatively recently this voltage limitation has not been a serious issue, but as the required capacity of submarine systems increases, it is becoming increasingly difficult to keep up with capacity demand subject to the power feed (PFE) constraint.

Every channel that is transmitted requires a certain optical signal power at the output of each amplifier in order to achieve the desired performance, and considering all the WDM channels, the total required amplifier output power is the sum of these channel powers. Even the most efficient amplifiers have a very limited power conversion efficiency (PCE), that is, the ratio of the amplifier optical output power to the supplied electrical power, and this figure is typically only 5%. Increasing the PFE voltage is not trivial as dielectric breakdown or creep within the cable and joints is difficult to overcome.

The required channel power for a given bit rate is lower now than for previous generations of technology due to the use of coherent receivers, advanced modulation formats and high performance forward error correction (FEC) schemes. Communication theory suggests that we are now close to the Shannon limit for a given wavelength channel and that any further improvements to fibre and transponders will only give us logarithmic increases in capacity. As a consequence, it has been recognised that significant increases in capacity will only be possible by choosing high spectral efficiency modulation formats or by increasing the number of channels. The easiest route to increase the number of channels for a given modulation format is by providing a larger transmission bandwidth, capable of supporting more WDM channels. Once the bandwidth is exhausted, space division multiplexing (SDM) is then necessary, either through multiple fibres, multiple fibre cores or mode division multiplexing. Significant advances have already being made in these areas [1], [2] and it is inevitable that we will reach the capacity limit due to the PFE voltage constraints once SDM technologies mature.

Here, the design methodologies which maximise the capacity of submarine systems subject to the PFE constraint are considered. Firstly the electrical powering conditions which maximise the power deliverable to the repeaters are reviewed. The optical system design is then optimised using the nonlinear Gaussian Noise (GN) model for optical transmission in two separate approaches.

In the first approach, for a system of given length, the optimum span loss is sought which minimises the required PFE voltage subject to the constraint that we wish to operate the system at the optimum channel power, that is, the channel power at which the balance of penalties due to amplified spontaneous emission (ASE) noise and nonlinearities result in a minimum bit error rate (BER).

In the second approach, the span loss which maximizes the capacity for a given PFE voltage and required optical signal to noise ratio (ROSNR) is sought. It is noteworthy that these two strategies were recently investigated in the context of minimising the electrical power rather than supply voltage [3], [4] and that the resulting span losses so obtained are essentially the same as the results derived below.

These optimisation strategies are then used to show that optimum designs operate towards the linear region of the fibre. As a consequence it is demonstrated that the fibre parameters which are traditionally tailored to minimise the fibre nonlinearity, such as nonlinear effective area and dispersion offer only a marginal improvements in system capacity. Nonlinear compensation techniques similarly offer little advantage, which simplifies the transponder DSP design. Finally, it is shown that higher order modulation formats use the available power inefficiently and that it is very hard to exceed capacities achievable using PM-QPSK. All these conclusions stem from the fact that for ULH submarine systems, the available power is limited and that the use of large bandwidth amplifiers and SDM allow us to avoid exhausting the required bandwidth.

# II. MAXIMIZING THE POWER DELIVERABLE TO THE REPEATERS

Consider the submarine line shown in Fig. 1 The PFE delivers a current I to  $n_r$  repeaters, each with a potential difference of  $V_r$ . If the submarine cable of length L has a span resistance  $R_s$ , a total resistance of  $R_c = n_s R_s$ , and the power dissipated by each repeater is  $P_r = IV_r$  then the required PFE voltage for the system is

$$V_{PFE} = IR_c + n_r \frac{P_r}{I} \tag{1}$$

where the number of spans  $n_s = n_r + 1$ . The potential



Fig. 1. PFE Configuration delivering power to  $n_r$  repeaters and cable

difference between the cable and the sea should be held below  $\sim 12 \text{ kV}$  to prevent creep and flash-over. For a given power requirement  $P_r$  of each repeater, the PFE voltage in (1) can be minimized with respect to current with the result that the optimum current is given by

$$I_{opt} = \sqrt{\frac{n_r P_r}{R_c}} \tag{2}$$

The effective repeater resistance  $R_r$  is given by  $P_r = I_{opt}^2 R_r$ . If we use this in (2) we find that the total cable resistance  $R_c$  and the repeater resistance are simply related by

$$R_c = n_r R_r \tag{3}$$

This is the familiar maximum power transfer theorem which states that for maximum power transfer, the source impedance (the total cable resistance in this case) and the load resistance (the sum of the resistances of the repeaters) should be equal. It follows that the total voltage drop across the cable should be equal to the total voltage drop across all the repeaters. A plot of PFE voltage versus current using (1) for a typical system is shown in Fig. 2. The figure clearly illustrates that if we are to maximise the power deliverable to the repeaters, operating at a low current can be just as inefficient as using a high current.



Fig. 2. PFE Voltage versus current for a ULH system illustrating the existence of a minimum voltage condition

Substituting the optimum current into (1), the minimum PFE voltage can be expressed as

$$V_{PFE}^{min} = 2\sqrt{R_c n_r P_r} \tag{4}$$

Having established the electrical maximum power transfer condition, it remains to choose the number of repeaters which maximizes performance while keeping the PFE voltage to a minimum. Examining this equation, it appears that a simple way to reduce the PFE voltage is to reduce the number of repeaters. However, the optimum optical channel power increases with the span length and so  $P_r$  increases as  $n_r$  decreases. As a consequence it is expected that there will be an optimum span length. This optimum is derived in the next section.

The other term in (4), the cable resistance, appears under the square root and so reducing the cable resistance is not very effective in reducing the PFE voltage.

#### III. DETERMINING THE OPTIMUM SPAN LENGTH AT THE OPTIMUM PERFORMANCE CONDITION

The electrical power required by the repeater and the optical signal power per channel  $P_{ch}$  are related according to

$$P_r = \frac{2n_{fp}n_{ch}P_{ch}}{\eta_c} + P_0 \tag{5}$$

where  $n_{fp}$  is the number of fibre pairs

- $n_{ch}$  is the number of bidirectional WDM channels per fibre pair
- and  $\begin{array}{c} \eta_c \\ P_0 \end{array}$  is the power conversion efficiency. and  $P_0$  is the power consumption of any control or other circuitry which is not related to optical power conversion.

The factor of 2 in (5) appears because the expression is for a set of bidirectional amplifiers but  $P_{ch}$  represents the unidirectional channel power. The power conversion efficiency  $\eta_c$  between the electrical amplifier power and the total optical output power can be as low as 1.2% [5], though it is possible to improve on this using modern techniques.  $\eta_c$  will depend on the amplifier technology and design and is treated here as a constant with a value of 5% throughout. Note, however, that it may not be possible to achieve this PCE for very wide band amplifiers due, for example, to increased GFF losses.

In an uncompensated nonlinear system with lumped amplifiers, the optimum channel power is given by[6]

$$P_{ch,opt} = B_s \sqrt[3]{\frac{(FG-1)h\nu}{2an_s^{\epsilon}}} \tag{6}$$

where a is the ratio of the single span nonlinear interference power spectral density (NLI PSD) to the signal PSD and is given by

$$a = \left(\frac{2}{3}\right)^{3} \gamma^{2} L_{eff}^{2} \frac{\sinh^{-1}\left(\frac{1}{2}\pi^{2} |\beta_{2}| L_{eff,\alpha} B_{WDM}^{2}\right)}{\pi |\beta_{2}| L_{eff,\alpha}}$$
(7)

This definition of a is chosen as it is independent of channel power and noise figure. However, a varies with span loss through  $L_{eff}$  and so this has to be taken into account in the derivation of the optimum span loss. The symbols in (7) are given by

$$\begin{split} L_{eff} &= \frac{1 - e^{-2\alpha L/n_s}}{2\alpha} & \text{effective length} \\ L_{eff,\alpha} &= \frac{1}{2\alpha} & \text{asymptotic nonlinear effective length} \\ \beta_2 &= -\frac{\lambda^2}{2\pi c} D & \text{dispersion coefficient with respect to frequency} \\ D & \text{dispersion coefficient with respect to wavelength } \lambda \end{split}$$

$$\begin{split} B_{WDM} &= B_s n_{ch}^{B_s/\Delta f} & \text{bandwidth of the whole WDM} \\ & \text{comb after removing the gaps} \\ & \text{between channels. For Nyquist} \\ & \text{spacing, } B_{WDM} = B_s. \\ & \epsilon \\ & \Delta f & \text{channel spacing} \end{split}$$

We now wish to minimise the PFE voltage with respect to the number of spans. From (4) we have

$$\frac{dV_{PFE}}{dn_s} = \frac{dV_{PFE}}{dn_r} = \frac{1}{2}V_{PFE}\left(\frac{1}{n_r} + \frac{1}{P_r}\frac{dP_r}{dn_r}\right) \tag{8}$$

where differentiability is assured by allowing  $n_r$  and  $n_s$  to take on real rather than integer values. At the minimum we obtain

$$\frac{dP_r}{dn_r} = -\frac{P_r}{n_r} \tag{9}$$

From (5), we can write

$$\frac{dP_r}{dn_r} = \frac{2n_{fp}n_{ch}}{n_c}\frac{dP_{ch}}{dn_r} \tag{10}$$

and combining (5), (9) and (10) we obtain

$$\frac{dP_{ch}}{dn_r} = -\frac{P_{ch}}{n_r} \left(\frac{P_r}{P_r - P_0}\right) \tag{11}$$

## A. Optimum span loss at the optimum channel power

If we operate at the optimum channel power then differentiating (6) we have

$$\frac{dP_{ch,opt}}{dn_r} = \frac{h\nu B_s^3}{6P_{ch}^2} \frac{d}{dn_s} \left(\frac{FG-1}{an_s^\epsilon}\right)$$
(12)

To evaluate the derivative on the right, the gain must be expressed in terms of the number of spans. By identifying the gain with the span loss it is given by  $G = e^{2\alpha L/n_s}$ . Also we note that the dependence of a on  $n_s$  is through  $L_{eff}$  alone. Performing this differentiation and eliminating  $dP_{ch}/dn_r$  in (11) and (12) by identifying  $P_{ch,opt}$  with  $P_{ch}$  in (11) we obtain

$$\ln G \approx 3\left(1 + \frac{P_0}{P_r}\right)\left(1 + \frac{1}{G}\left(2 - \frac{1}{F}\right)\right) \tag{13}$$

This is essentially equivalent to the result  $\ln G = 3$ , or  $10 \log_{10} G = 13 \,\mathrm{dB}$  derived in [3], to within the approximations employed, where here, the voltage rather than power is minimised. The span loss is fairly insensitive to the noise figure but the fractional electrical power overhead  $P_0/P_r$  is significant. Furthermore, the result is independent of PFE voltage, conversion efficiency, ROSNR and nonlinear parameters. The gain (span loss) is illustrated in Fig. 3 for F = 3 (4.8 dB) where the effect of the power overhead  $P_0$  is expressed as a percentage of the repeater power  $P_r$ .



Fig. 3. Optimum span loss versus percentage power overhead  $P_0/P_r$  at the optimum channel power

To illustrate how this optimum condition might be used, consider the design of an 11000 km  $100 \times 100 \text{ Gb/s}$  PM-QPSK WDM system with Nyquist channel spacing using a

Parameter	Symbol	Value
Attenuation Coefficient (dB)	α	0.16
Dispersion Coefficient (ps/nm-km)	D	20
Nonlinear Effective Area $(\mu m^2)$	$A_{eff}$	130
Coherence Factor	$\epsilon$	0.07
Noise Figure (dB)	F	4.5
FEC Overhead (%)	OH	25
Power Conversion Efficiency(%)	$\eta_c$	5
Cable Resistance (Ω/km)	$r_o$	1
Power Overhead (%)	$P_0/P_r$	10

 
 TABLE I.
 System design parameters used to optimise the design assuming operation at the optimum channel power

power overhead  $P_0/P_r$  of 10%. Then the optimum span loss is determined to be 15.1 dB. The other system parameters are given in Table I. Furthermore, let us suppose that the FEC threshold is 11 dB and that we require the system margin, after taking into account the nonlinear penalties, to be 2.5 dB above the FEC threshold, i.e., we require a nonlinear OSNR of 13.5 dB. In Fig. 4 the OSNR is plotted versus channel power for the system. Considering that the optimum span loss condition does not fix the OSNR at the receiver, it is quite fortuitous that the nonlinear OSNR comes out at 13.75 dB, 0.25 dB above the ROSNR.

The lack of control of the OSNR for this optimisation regime is problematic. To illustrate this, suppose we designed a shorter system than in this example. The optimum span loss would be unchanged and so the OSNR at the receiver will increase, resulting in a greater margin than necessary. One way round this is to change the baud rate to better match the OSNR, but this is not always possible.



Fig. 4. Adjusting the channel power to suit the ROSNR

Another alternative to overcome the mismatch between the OSNR and ROSNR is to reduce the channel power. Examining Fig. 4 we see that when the nonlinear OSNR reduces to the required 13.5 dB, the channel power reduces from -2.2 dB to -3.2 dB, a drop of 1dB. This relatively large drop is due in the most part to the fall in nonlinear penalty by 0.8 dB. This drop in signal power allows us to increase the number of channels by a factor of 1.2 for the same PFE voltage.

Modifying the channel power in this way takes us away from the optimum conditions but it suggests an alternative optimization strategy, that is, to maximise the capacity for a specified OSNR and PFE voltage. This methodology is applied in the next section.

#### B. Optimum span loss for maximum system capacity

In this section we seek to maximize the number of channels subject to the constraint that we wish to achieve a given nonlinear OSNR at a specified PFE voltage. From (4) and (5) we can write

$$n_{ch} = \left(\frac{V^2}{4R_c n_r} - P_0\right) \frac{\eta_c}{2n_{fp} P_{ch}}.$$
 (14)

Maximising  $n_{ch}$  with respect to  $n_r$  yields the condition

$$\frac{n_r}{P_{ch}}\frac{dP_{ch}}{dn_r} = \frac{-1}{(1 - P_0/P_r)}.$$
(15)

which is equivalent to (11). To evaluate  $dP_{ch}/dn_r$  we use the nonlinear OSNR according to the GN model which can be written

$$O_{GN} = \frac{P_{ch}}{B_N \left[ n_r \left( F e^{2\alpha L/n_s} - 1 \right) h\nu + \frac{a n_s^{(1+\epsilon)} P_{ch}^3}{B_s^3} \right]}$$
(16)

In this case we do not insist that we operate at the optimum power (6). Differentiating (16) with respect to  $n_r$  for a fixed nonlinear OSNR and substituting in (15) then yields, after some manipulation,

$$\ln G \approx 2\left(1 - \frac{1}{FG}\right)\left(1 + \frac{P_0}{2P_r} - \frac{1}{2}\left(\frac{P_{ch}}{P_{ch,opt}}\right)^3\right) \quad (17)$$

Again, this is similar to the result obtained in [4], but in this case we include the effect of operating at a specified channel power relative to the optimum channel power. For channel powers a few dB below the optimum defined by (6) this yields a span loss of in the range 8 – 9 dB. This is illustrated below in Fig. 5. There is clearly a significant penalty in choosing a channel power close to the optimum as the span loss becomes impractically small. In terms of the nonlinear penalty  $\Delta P_{nl} = 10 \log_{10} (O_{GN}/O_{lin})$ , a channel power 3dB below the optimum channel power corresponds to a nonlinear penalty of only 0.26 dB where we note that the linear OSNR  $O_{lin}$  and nonlinear OSNR  $O_{GN}$  are simply related according to

$$\frac{1}{2} \left( \frac{P_{ch}}{P_{ch,opt}} \right)^3 = \frac{O_{lin}}{O_{GN}} - 1.$$
(18)

#### IV. DESIGN EXAMPLES

Consider again the design of the transpacific link defined above in section III-A. If we operate at the optimum channel power of -2.2 dBm, assuming Nyquist channel spacing, then we find from (4) and (5) that if we allow 8 fibre pairs then 12 kV is reached if we have 132 channels per fibre pair occupying 37 nm. The total system capacity is 111 Tb/s. Operating at a lower channel power with the same span loss in order to achieve a nonlinear OSNR of 13.5 dB allows us to



Fig. 5. Optimum span loss versus channel power relative to the optimum channel power for a power overhead of 0%, 10% and 20%

increase the capacity by a factor of 1.2, as mentioned above, to 133 Tb/s.

If we now maximise the system capacity according to the strategy in III-B subject to the constraint that the maximum PFE voltage is 12 kV and that we require the same system margin we find that the optimum span loss is 8.5 dB (slightly higher than the approximate value obtained from (17) which corresponds to a span length of 53 km. The corresponding channel power is -8.4 dBm, over 6 dB less than above and the total system capacity is 257 Tb/s, over double what could be achieved by minimising the PFE voltage alone. The channels require 16 fibre pairs at 43 nm per fibre pair.

The sensitivity of the capacity to span loss is illustrated in Fig. 6. It can be seen that there is little drop in capacity by operating the system with 10 dB spans. The corresponding reduction in the number of repeaters is a cost efficient compromise.



Fig. 6. System capacity versus span loss for a fixed PFE voltage of  $12 \, kV$  and  $13.5 \, dB$  nonlinear OSNR.

#### V. DISCUSSION

By operating the system at a channel power 6.2 dB less than the optimum, the nonlinear penalty is only than 0.03 dB. This suggests that we do not need to be overly concerned with the nonlinear effective area or dispersion and therefore nonlinear mitigation. If we analyse the impact of operating the system with a fibre of  $A_{eff} = 80 \,\mu\text{m}^2$  and  $D = 17 \,\text{ps/nm-km}$ , then if we retain the same span loss we can achieve a system capacity of 237 Tb/s. This is a relatively small change in capacity for such a large change in effective area. Again, this may be an attractive option to reduce cable costs. The variation of capacity with effective area is illustrated in Fig. 7 for a fixed dispersion of 20 ps/nm-km.

One advantage of using pure silica core (PSC) fibre with a standard effective area is that distributed Raman amplification becomes more efficient because the Raman pump power decreases linearly with effective area. This is an attractive option for wide band hybrid amplifiers compared with C+L band EDFAs because, although the PCE is generally lower for hybrid amplifiers, the repeater architecture is simpler and the average noise figure is lower. The use of low attenuation fibre is also of great benefit as it both increases the span lengths and allows the Raman pump to penetrate further into the fibre.



Fig. 7. System capacity versus nonlinear effective area for a fixed PFE voltage of 12 kV and 13.5 dB nonlinear OSNR.

In the example given, the repeater must support 16 fibre pairs, each with an amplifier bandwidth of 43 nm. An alternative would be to use 10 fibre pairs with 68 nm amplifiers. This would save on cable costs, would simplify the repeater design and highlights the importance of large bandwidth amplifiers in contemporary designs. The PM-QPSK modulation format has been assumed as it is power efficient compared with higher order modulation formats. For example, if we doubled the bit rate using PM-QPSK to 200 Gb/s, i.e., double the baud rate, we would need to increase the ROSNR by 3 dB. The optimum channel power increases by 3dB and there is no change in the system capacity, and little change to the required bandwidth. However, if we use PM-16QAM, for example, to achieve the same bit rate then the ROSNR must increase by approximately 6.5 dB, a 3.5 dB penalty over PM-QPSK. Examining this scenario we find that the optimum span loss becomes extremely short to support the increased ROSNR. If however, we reduce the ROSNR threshold from 14 dB to 13.3 dB by improved FEC techniques, we can achieve the same 257 Tb/s capacity at half the bandwidth but the span loss reduces to 7.2 dB. By contrast, if we apply the same 13.3 dB ROSNR to PM-QPSK, the capacity increases to 305 Tb/s and the optimum span loss increases marginally to 8.6 dB. For

ULH systems where bandwidth is not a constraint, there is no need to use higher order modulation formats. For shorter systems where the PFE is no longer a constraint and bandwidth is more difficult to achieve, higher order modulation formats become advantageous.

Although the optimum span loss is insensitive to most system parameters, the achievable capacity is highly dependent on them. In particular, the total capacity varies linearly with PCE and noise figure. Furthermore, the capacity varies as the square of the PFE voltage, so if, in the future, cable and joint high voltage performance is improved, then we can expect to see still higher capacities attainable over transpacific distances.

## VI. CONCLUSION

Methodologies have been investigated to maximise the capacity of ULH submarine systems by optimising the power delivered to the repeaters through matching the total repeater voltage with the cable voltage drop and by optimizing the optical span loss. It has been found that optimising the span losses to minimize the system voltage yields the same result as for minimising the system power with a typical value of 15 dB. This result is independent of system length and so there can be a large mismatch between the required and achieved OSNR. However, if we alternatively maximise the system capacity for a specified ROSNR and system voltage we find the optimum span loss is approximately 8.5 dB. With little reduction in capacity, the span loss can be increased to 10 dB. For ultralow loss (ULL) fibre, this corresponds to span lengths of 53 km and 63 km respectively. Optimizing the span loss for maximum capacity results in channel powers significantly below the optimum channel power so that the system is operating in the linear regime.

As a result, we can achieve excellent performance by choosing ultra-low loss fibre with  $80 \,\mu m^2$  and 17 ps/nm-km dispersion. The ideal modulation format for ULL submarine systems is PM-QPSK as there is a loss of capacity by choosing higher order formats. The maximum capacity achievable over transpacific distances is found to be 250 Tb/s. To achieve this capacity it is essential that large bandwidth amplifiers are employed to keep the number of fibre cores and fibre pairs to a minimum. Using 10 SDM channels, the required amplifier bandwidth is 70 nm.

#### REFERENCES

- [1] K. Shibahara, T. Mizuno, H. Takara, A. Sano, H. Kawakami, D. Lee, Y. Miyamoto, H. Ono, M. Oguma, Y. Abe, T. Kobayashi, T. Matsui, R. Fukumoto, Y. Amma, T. Hosokawa, S. Matsuo, K. Saitoh, H. Nasu, and T. Morioka, "Dense SDM (12-core X 3-mode) Transmission over 527 km with 33.2-ns Mode-Dispersion Employing Low-Complexity Parallel MIMO Frequency-Domain Equalization," in *Optical Fiber Communication Conference Post Deadline Papers*. Optical Society of America, 2015, p. Th5C.3. [Online]. Available: http://www.osapublishing.org/abstract.cfm?URI=OFC-2015-Th5C.3
- [2] K. Igarashi, T. Tsuritani, I. Morita, and M. Suzuki, "Ultra-long-haul highcapacity super-nyquist-wdm transmission experiment using multi-core fibers," *Lightwave Technology, Journal of*, vol. 33, no. 5, pp. 1027–1036, March 2015.
- [3] Doran, N.J., I. and Ellis, A.D., "Optical link design for minimum power consumption and maximum capacity," *Optical Communication (ECOC* 2013), 39th European Conference and Exhibition, sept. 2013.
- [4] —, "Minimising total energy requirements in amplified links by optimising amplifier spacing," *Opt. Express*, vol. 22, pp. 19810–19817, 2014. [Online]. Available: http://www.opticsinfobase.org/oe/abstract.cfm?URI=oe-22-16-19810
- [5] T. Frisch and S. Desbruslais, "Electrical power, a potential limit to cable capacity," in *Suboptic*, 2013., April 2013, paper Tu 1C 04.
- [6] Poggiolini, P., "The GN Model of Non-Linear Propagation in Uncompensated Coherent Optical Systems," *Lightwave Technology, Journal of*, vol. 30, no. 24, pp. 3857–3879, Dec 2012.